**QUIZ 5 Solutions** **MATH 310**

PROBLEM 1

Find polynomials q(x) and r(x) such that f(x) = g(x)q(x) + r(x), and r(x) = 0 or $degr(x) < degg(x)$:
(a) f(x) = $3x^4 - 2x^3 + 6x^2 - x + 2$ and $g(x) = x^2 + x + 1$ in $\mathbb{Q}$[x]

f(x)=$(x^2 + x + 1)(3x^2 - 5x + 8) + (-4x - 6)$

(b) f(x) = $2x^4 + x^2 - x + 1$ and $g(x) = 2x - 1$ in $\mathbb{Z}_5[x]$

f(x)=$(2x - 1)(x^3 + 3x^2 + 2x + 3) + 4$

PROBLEM 2

If R has multiplicative identity $1_R$, show that $1_R$ is also the multiplicative identity of R[x].

proof: For any element a = $a_0 + a_1x + ... + a_nx^n \in R[x]$

$1_R * a = 1_R$ * ( $a_0 + a_1x + ... + a_nx^n$) = $1_R * a_0 + 1_R * a_1x + ... + 1_R * a_nx^n$

=$a_0 + a_1x + ... + a_nx^n$ = a

$a * 1_R$ = ( $a_0 + a_1x + ... + a_nx^n$) * $1_R$ = $a_0 * 1_R + a_1x * 1_R + ... + a_nx^n * 1_R$

=$a_0 + a_1x + ... + a_nx^n$ = a

Thus, a = $1_R * a = a * 1_R$

So $1_R$ is also the multiplicative identity of R[x].